

NOISE CANCELLATION BY DIGITAL ADAPTIVE FILTER BASED ON NLMS ALGORITHM

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ABSTRACT

Adaptive filters are best techniques used in cases where signal conditions or system parameters are slowly changing and the filter is to be adjusted to compensate for this change. Adaptive filters are very much used in variety of signal processing applications such as system identification, interference cancellation and linear prediction. Particularly here, the design of the adaptive filter will be for noise cancellation for noisy signal. Adaptive noise cancellation is a technique of estimating signals corrupted by additive noise. This literature involves on the removal of the noise corrupting the signal using adaptive noise cancellation algorithms. The algorithm is implemented using MATLAB.

Keywords: Adaptive Filters, NLMS Algorithm, Noise Cancellation.

1.INTRODUCTION.

The one of the most important subject in signal processing and communications field is to remove the noise and distortion from the signals. Adaptive filter systems offers the possibilities for achieving this goal . One effective technique to remove noise is Adaptive Noise Cancellation (ANC). An adaptive system is self-adjusting and is a time varying and nonlinear system. The basic concept of ANC is to pass the noised signal through the ANC which consists of a digital filter which cancels the noise signal and passes the desired signal . This process is an adaptive process, the advantage

of it is , that prior knowledge of the signal and noise characteristics is not necessary. Different adaptive filter algorithms are used for noise cancellation e.g. least mean square (LMS) algorithm, the recursive least square (RLS) algorithm , etc .The LMS algorithm is most popular due to its simplicity. Apart from this, many variants of LMS algorithm such as Normalized LMS (NLMS) algorithm, Variable Step-size (VSSLMS) algorithm, Block LMS (BLMS) algorithm are implemented for Adaptive filter systems applications. The NLMS adaptive algorithms technique is chosen to be used in the current work to provide the strategy for adjusting the filter coefficients to estimate the original signal. The proposed optimization algorithm is simulated using MATLAB programming to offer the possibilities for better performance and efficiency.

2. NORMALIZED LEAST MEAN SQUARE ALGORITHM (NLMS)

To derive the normalized least mean square algorithm (NLMS) algorithm we consider the standard LMS recursion, for which we select a variable step size parameter, $\mu(n)$. This parameter is selected so that the error value, $e^+(n)$, will be minimized using the update filter tap weights, $w(n+1)$, and the current input vector, $x(n)$.

$$w(n+1) = w(n) + 2\mu e(n)x(n) \quad (2.1)$$

$$e^+(n) = d(n) - w^T(n+1)x(n) \quad (2.2)$$

$$= (1 - 2\mu(n)x^T(n)x(n))e(n) \quad (2.3)$$

Next we minimize $[e(n)]^2$, with respect to $\mu(n)$. Using this we can find a value for $\mu(n)$ which forces $e^+(n)$ to zero.

$$\mu(n) = \frac{1}{2x^T(n)x(n)} \quad (2.4)$$

This $\mu(n)$ is then substituted into the standard LMS recursion replacing $\mu(n)$, resulting in the following .

$$w(n+1) = w(n) + 2\mu e(n)x(n) \quad (2.5)$$

$$w(n+1) = w(n) + \frac{1}{x^T(n)x(n)} e(n)x(n) \quad (2.6)$$

2.1 Implementation of The NLMS Algorithm

As the NLMS is an extension of into the standard LMS algorithm , the NLMS algorithm implementation is very similar to that of the LMS algorithm. Each iteration of

the NLMS algorithm requires these steps in the following order.

Calculate the output of the adaptive filter.

$$y(n) = \sum_{i=0}^{N-1} w(n)x(n-i) = w^T(n)x(n) \quad (2.7)$$

Calculate an error signal as the difference between the desired signal and the filter output.

$$e(n) = d(n) - y(n) \quad (2.8)$$

Calculate the step size for the input filter

$$\mu(n) = \frac{1}{x^T(n)x(n)} \quad (2.9)$$

The filter tap weights are updated in preparation for the next iteration.

$$w(n+1) = w(n) + \mu(n)e(n)x(n) \quad (2.10)$$

Each iteration of the NLMS algorithm requires $3N+1$ multiplications, this is only N more than LMS algorithm [2].

2.2 Adaptive Digital Systems:

An adaptive signal processing system is a system which has the ability to change its processing behavior in a way to maximize a given performance measure. An adaptive system is self-adjusting and is, by its nature, a time varying and non-linear system. Figure 2.1 shows a generic adaptive signal processing system.

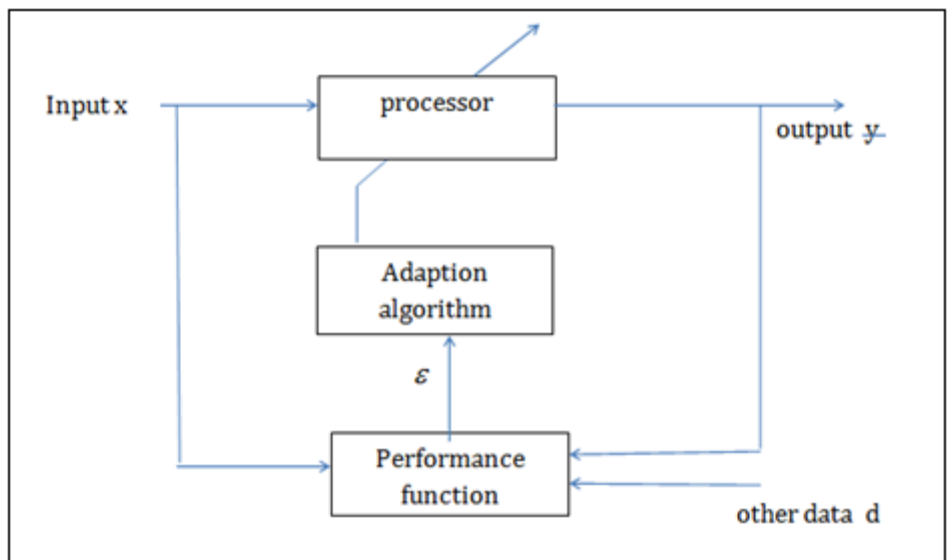


Figure 2.1 A generic adaptive system

In the most general form, the adaptive linear combiner is assumed to have $L+1$ inputs denoted by

$$x_k = [x_{0k} \ x_{1k} \ \dots \ \dots \ \dots \ x_{lk}]^T \quad (2.11)$$

Where k is used as the time index.

Similary, the weights are denoted :

$$w_k = [w_{0k} \ w_{1k} \dots\dots\dots w_{lk}]^T \quad (2.12)$$

Since we are now dealing with an adaptive system, the weights will also vary in time and hence have a time subscript k . The output y can be expressed as

$$y_k = \sum_{l=0}^L w_{Lk} x_{lk} \quad (2.13)$$

$$\text{or} \quad y_k = x_k^T w_k = W_k^T X_k \quad (2.14)$$

The adaptive linear combiner can also take another form with only a single input as shown in figure 4.2.

In the case the output is given by the convolution sum

$$y_k = \sum_{l=0}^L w_{Lk} x_{k-l} \quad (2.15)$$

In the case the output is given by the convolution sum

$$y_k = \sum_{l=0}^L w_{Lk} x_{k-l} \quad (2.16)$$

Which is the expression for the FIR filter or transversal filter [5] .

The performance function will be based on the error signal e_k as shown in figure 2.2

$$e_k = d_k - y_k \quad (2.17)$$

$$e_k = d_k - x_k^T w = d_k - w^T x_k \quad (2.18)$$

Now it takes the square of equation (2.18), it got

$$\begin{aligned} e_k^2 &= (d_k - w^T x_k)(d_k - x_k^T w) \\ &= d_k^2 + w^T x_k x_k^T w - 2d_k x_k^T w \end{aligned} \quad (2.19)$$

Assume that e_k , d_k and x_k are statistically stationary and take the expected value of equation (2.19) over k to obtain the MSE

$$\eta = E[e^2] = E[d_k^2] + w^T E[x_k x_k^T] w - 2E[d_k x_k^T] w \quad (2.20)$$

$$= E[d_k^2] + w^T R w - 2P^T w \quad (2.21)$$

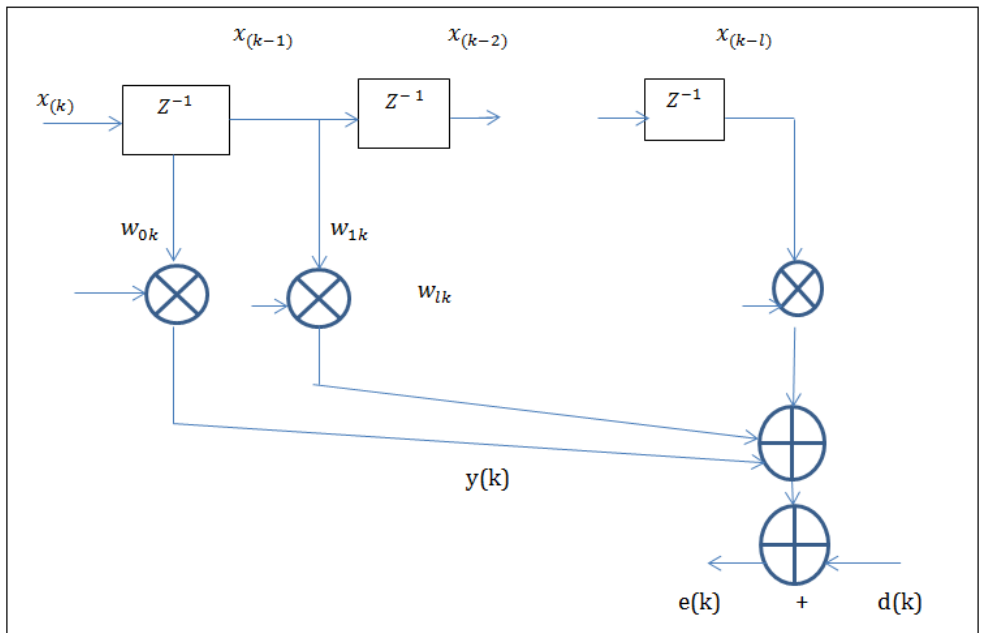


Figure 2.2 Adaptive filter block diagram.

Where the matrix R is the square input correlation matrix .

$$R = E[x_k x_k^T] = E \begin{bmatrix} x_{0k}^2 & x_{0k}x_{1k} \cdots & x_{0k}x_{Lk} \\ x_{1k}x_{0k} \vdots & x_{1k}^2 \ddots \cdots & x_{1k}x_{Lk} \vdots \\ x_{Lk}x_{0k} & x_{Lk}x_{1k} \cdots & x_{Lk}^2 \end{bmatrix} \quad (2.22)$$

The vector P is the cross-correlation between the desired response and the input components.

$$P = [d_k x_{0k} d_k x_{1k} \dots \dots d_k x_{Lk}] \quad (2.23)$$

Differentiating equation 2.21 to get the gradient

$$\nabla(\eta) = \frac{\partial \eta}{\partial w} = \left[\frac{\partial \eta}{\partial w_0} \frac{\partial \eta}{\partial w_1} \dots \dots \frac{\partial \eta}{\partial w_L} \right]^T = 2RW - 2P \quad (2.24)$$

The optimum weight vector is found where the gradient is zero, hence

$$\nabla(\eta) = 0 = 2RW - 2P \quad (2.25)$$

Assuming that R is non-singular, the wiener-Hopf equation in matrix form is

$$W^* = R^{-1}P \quad (2.26)$$

From this we realize that adaptation (optimization); i.e. finding the optimum weight vector, is an iterative method of finding the inverse of the input correlation matrix.

The minimum mean square error (at optimum) is now obtained by substitution W^* from equation 2.26 for W in equation 2.21 and using the symmetry of the input correlation matrix, i.e. $R^T = R$ and $(R^{-1})^T = R^{-1}$

$$\begin{aligned} \eta_{min} &= E[d_k^2] + \dot{W}^T R \dot{W} - 2P^T \dot{W} \\ &= E[d_k^2] + (R^{-1}P) R R^{-1}P - 2P^T R^{-1}P \end{aligned}$$

$$\begin{aligned}
 &= E[d_k^2] - P^T R^{-1} P \\
 &= E[d_k^2] - P^T \hat{W}
 \end{aligned}
 \tag{2.27}$$

3. SIMULATION OF ADAPTIVE FILTERING NLMSALGORITHM USING MATLAB

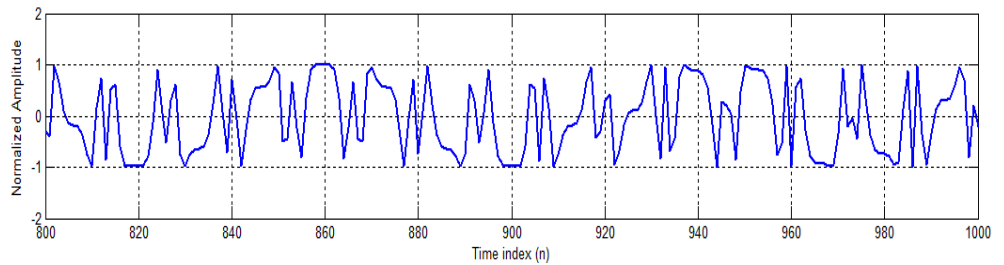
In this experiment the speech signal is mixed with the white Gaussian noise and then passed through the NLMS algorithm based filter to get the desired output. The NLMS filter updates its coefficients until the noise from the mixed signal is removed. By changing the filter length and the S/N ratio in dB to get the minimum value of the error $e(n)$ which is the difference between the desired signal, $d(n)$ and the adaptive filter output, $y(n)$.

TABLE (3.1) COMPARISON OF THE MSE VALUE

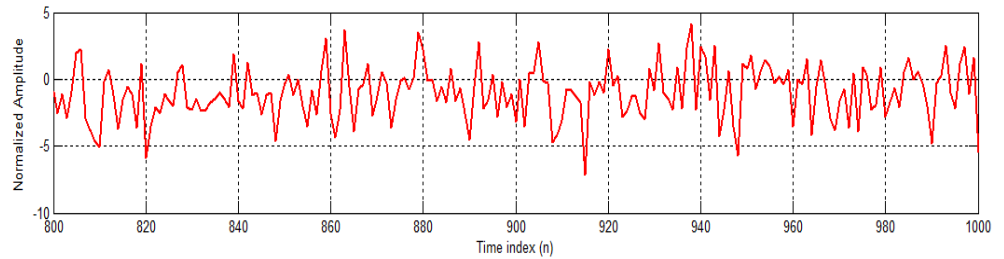
FILTER LENGTH	FILTER LENGTH		
	L=20		
SNR in dB	10	20	30
NLMS MSE VALUE	0.5542	0.2559	0.0584

By using the NLMS algorithm for LPC adaptive system it has noted that the MSE value as a general is smaller than the LMS algorithm and gives better performance and faster convergence and it is obtained the smallest error when SNR = 30dB (MSE = 0.0401) and the largest value when SNR=10 (MSE= 0.6380). Therefore, the NLMS algorithm offers fast convergence toward the optimum solution specially when the SNR value is high. Other

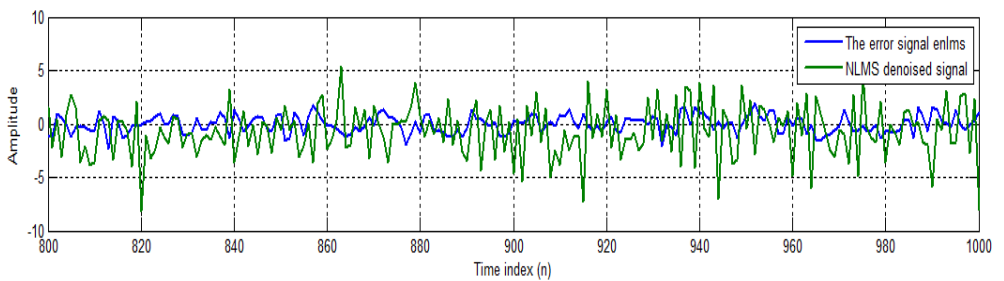
filter lengths can be used like $L=20$, 30 to minimize the **MSE VALUE** and to remove the noise corrupting the original signal .



(a)



(b)



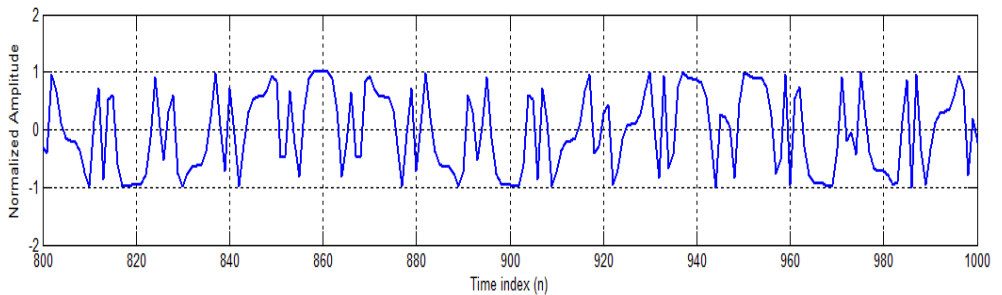
(c)

Figure 3.1 NLMS algorithm output for $L=20$, $SNR=10dB$

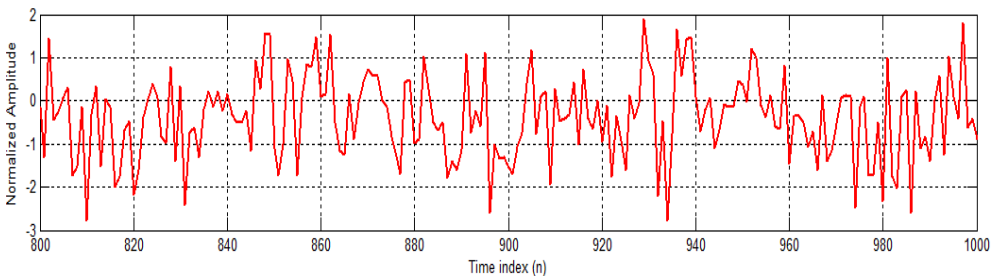
a) Original signal.

b) Original signal+noise (noised signal).

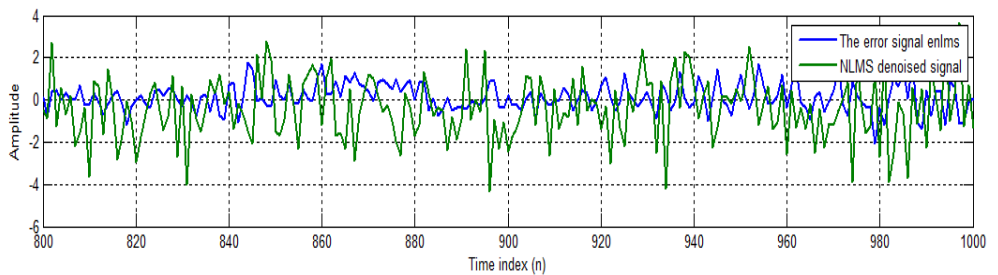
c) The output of the adaptive NLMS algorithm (NLMS denoised signal) and the error signal which is the difference between the noised signal and the input signal



(a)



(b)



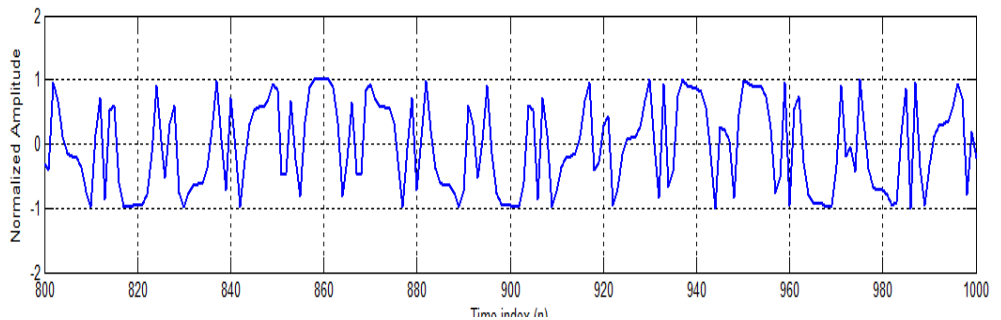
(c)

Figure 3.2 NLMS algorithm output for $L=20$, $SNR=20dB$

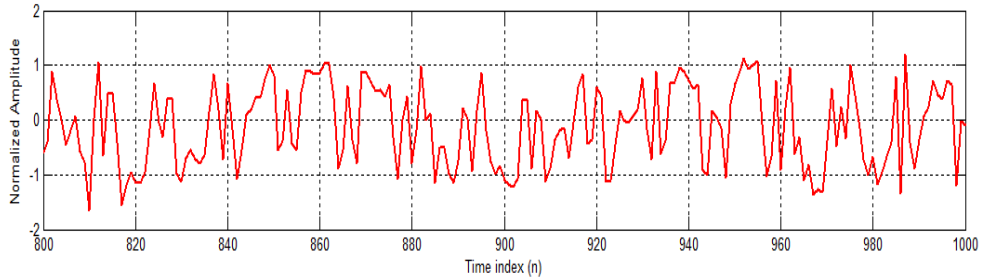
a) Original signal.

b) Original signal+Noise (noised signal).

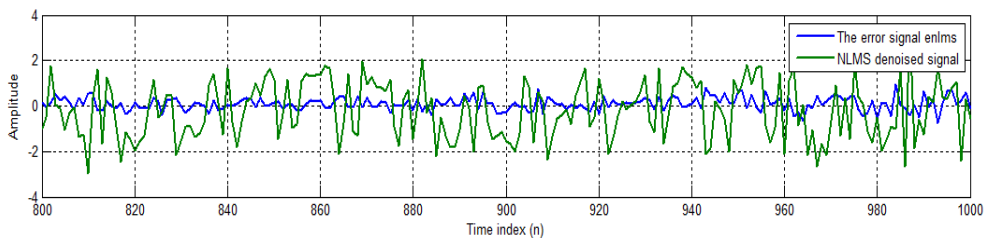
c) The output of the adaptive NLMS algorithm (NLMS denoised signal) and the error signal which is the difference between the noised signal and the input signal.



(a)



(b)



(c)

Figure 3.3 NLMS algorithm output for $L=20$, $SNR=30dB$

- a) Original signal.
- b) Original signal+Noise (noised signal).
- c) The output of the adaptive NLMS algorithm (NLMS denoised signal) and the error signal which is the difference between the noised signal and the input signal.

4. CONCLUSION

The adaptive filtering algorithms are powerful tools that can be used in many applications. An adaptive filter is a procedure for adjusting the parameters of an adaptive filter to minimize a cost function. A highly popular cost function is the mean square error criterion, defined as the mean-square value (MSE) of an estimation error, denoted by $e(n)$. Clearly, the smaller the estimation error, the better the filter performance. As the error approaches zero, the output of the filter approaches the original signal. Only an adaptive FIR filter structure has been considered.

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