Journal of Alasmarya University: Applied Sciences
Vol. 10, No. 3, Sep 2025, 195-214

مجلة الجامعة الأسمرية: العلوم التطبيقية
المجلد 10، العدد 3، سبتمبر 2025، 195- 214

https://doi.org/10.59743/jau.v10i3.2194

Development of a mathematical model for the selection of the Project Delivery Method for the Libyan Public Construction Projects

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Abstract

The selection of a project delivery method (PDM) for the implementation of any construction project is considered one of the most important and critical factors on which depends the successful completion of the project. Furthermore, the process by which the project delivery method will be selected can range from basic decisions to complicated and intricate processes. The current research focuses on the development of a simple mathematical model for the selection process based on the analytical hierarchy process (AHP) as a decision tool.

The main objectives of this paper are: introducing the concept of project delivery to the local construction industry; presenting the most common methods of the project delivery; and, developing a simple, yet practical and useful model for selecting the optimal project-specific delivery method.The research concluded that the most appropriate method for use, depending on the owner's preferences, was the construction management at risk (CM@R), which scored the highest rank with total of 0.53, followed by integrated project delivery (IPD) with total of 0.22. However, design-build (DB) and design-bid-build (DBB) both ranked the lowest with total scores of 0.14 and 0.11, respectively.

Keywords: Construction management; Highway Construction Project; Project delivery method; Decision Preferecnes, Analytical Hierarchy Process (AHP).

I. INTRODUCTION

Construction projects are characterized, mainly, by their time-consuming nature. Long activity durations, consecutive activity relations and sequential construction processes are typically associated with such projects, especially those of mega scale infrastructural ventures [1], [2]. The use of systematic procedures and scientific methods in the field of decision making and problem solving has grown noticeably in the last few decades [3]. Due to the growing complexity and increasing criticality of the environment surrounding the construction industry, decision and policy makers are searching for new and innovative techniques and procedures as well as adopting and implementing scientific and especially mathematical methods to the study and analysis of problems involving complex systems [2], [4]. Since early ages of the construction industry, a delivery method was manifested and

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ISSN: 2706-9524 (Print) ISSN: 2706-9532 (Online)

Received: 06 July 2025; Accepted: 29 September 2025; Available online: 7 October 2025

embodied in order to pursue the final product and achieve the desired objectives and the ultimate goal [3], [5], [6]. As the construction industry developed and progressed over the years so did the its delivery methods, originating from traditional approaches, such as design-bid-build delivery method, to a more sophisticated, refined methods that have the proficiency to overcome the shortcomings and drawbacks of the traditional methods, such as integrated project delivery (IPD), multi-prime (MP) and public-private-partnership (PPP) [1], [7]. Numerous delivery methods are available and being used in the industry, nonetheless, there is NO one method that can be used in delivering the same kind of projects with different preferences and objectives, let alone all sorts of projects. Understanding the importance and criticality of choosing the delivery method that best services the objectives of the project taking into consideration the preferences of its owner as well as the prospects of the its parties [1], [6], [8]–[16].
"A Project delivery method is a system for organizing and financing design, construction, operations and maintenance activities and facilitates the delivery of a good or service" [14], [15]. The project delivery method PDM contractually defines the structure of the relationships of the contracted parties, legally describes the roles, responsibilities and accountabilities of the project parties, and the general sequence of activities required to deliver the project [7], [13], [17].
As mentioned before, even for the same type of construction project, the appropriate project delivery method differs considerably depending on many factor; to name a few [4], [7]–[9], [12]–[14], [18]–[21]:

- Preferences of the owner,
- Experience of the owner,
- Constraints of the project,
- Criticality, seriousness and nature of the project,
- Technical and administrative constraint of the project,
- Environment of the project.

In order to select the optimal construction delivery method, a selection approach along with selection criteria need to be considered and developed [9], [22], [23]. The effective implementation of the designs and execution of the plans, and overall successful completion of the construction project depends on the selection of the optimal construction project delivery method. The suitability of the selected method for delivering the construction project relies on the identifying and determining driving criteria for the final objective as well as the establishment of a systematic and efficient selection approach [7], [9], [14], [15], [20], [24], [25]. Mitigated conflicts, alleviated risks, and decreased costs are some of the benefits and advantages gained from selecting the most appropriate delivery method [9], [23]. However, due to the time-consuming nature and future-dependency of construction projects as well as the numerous and varying uncertainties that surround the implementation and construction stage, the selection of the appropriate PDM is not an easy task. Identifying selection options, determining selection criteria and evaluating successfulness of the developed approach are critical steps for the selection of the optimal and most appropriate delivery method [7]–[10], [12], [13], [17], [18], [20].

II. LITERATURE REVIEW

Selection of the delivery method that best serves a given project requires knowledge of the nature and characteristics of the project as well as understanding of the project delivery methods. Identifying the main objectives of the construction project and determining criteria preferences as well as choosing a decision making tool are an important milestones of the selection process.

Many models for the selection process have been developed and presented in recent decades, qualitative and quantitative models were proposed, Gordon [12] presented a qualitative model of removal algorithm for selecting PDS, in which, the inappropriate system for construction project is removed. Al Khalil [26] developed a model for use in selecting the most appropriate delivery method using the analytic hierarchy process, a ranking of the delivery methods was obtained by considering several relevant factors. Guomin Zhang and Patrick X. Zou [27] used qualitatively based methodologies in analysing the risks and setting up a hierarchy structure of those risk, and used a fuzzy analytical hierarchy process (FAHP) model to appraise the risk environments associated with the joint ventures in order to support the decision making process. The proposed fuzzy AHP model was used on an empirical case study, which demonstrated effectiveness in dealing with the risks associated with joint venture projects.

Ali Mostafavi and Mohammad Karamouz [24] developed a fuzzy multi-attribute decision-making (FMADM) model by utilizing fuzzy approach in order to rank project delivery system alternatives and considering risk attitude of the decision maker and the lessons learned from previous experiences, apetrochemical project was used as a case study. Khalegh Barati et al [20], used the analytic hierarchy process for the selection of the optimal project delivery system in which a comprehensive study was performed on different infrastructural projects, the research proposed a contractual management software as tool for aiding in selecting optimal construction system. Xingyu ZHU et al. [8] developed a novel model for decision-making to be used in selecting a construction project delivery system based on case-based reasoning (CBR) and robust nonparametric production frontier method, a questionnaire survey was conducted and the validation of the model was tested

A. Project delivery methods (PDM*)*

Project delivery is a comprehensive and extensive process including planning, designing and constructing the final product, in order to complete the building facility with prescribed and predetermined requirements. choosing the delivery method that best serves a specific project is one of the fundamental and critical decisions that have to be made at some point. It is important that the owner of the project have an understanding of the importance of the delivery method and its criticality in the successful completion of the project, as well as, the available and commonly used methods which used with different types of projects, moreover, its essential that project owner develops selection criteria and prioritize their preferences regarding project requirements and constraints, as well as the level of involvement, risk exposure and the bearable responsibility [4], [6].

The choice of the delivery method depends on many factors and varies for the same project with different preferences [4]. Furthermore, the specific delivery method can be modified and revised to meet the specific needs of the owner and requirements of the project. There are several delivery methods that are used commonly in the construction industry, including design-bid-build (DBB), design-build (DB), construction management at risk (CM-at risk), also known as construction management as general contractor (GC), and integrated project delivery (IPD) [9].

However, in the current research, four of the main and commonly used delivery methods were considered and used as alternatives.

Design-did-build (DBB)

Also known as the traditional method, figure 1, a delivery method which typically involves three sequential phases; the design phase, in which the owner contracts with a designer for the services of designing the project; the bid phase, in which the owner selects and contracts with the contractor, and the build phase, in which the contractor executes the designs of the designer in order to construct the project. Usually, a sealed bid, fixed-price contract is associated with the BDD [1], [9], [18].

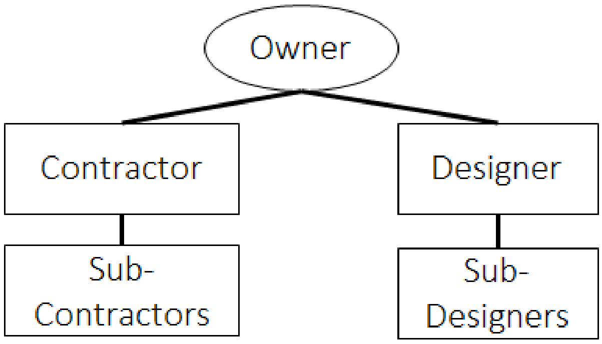

Fig. 1: Schematic representation of the Design-Bid-Build (DBB).

Design-build (DB)

A project delivery method which was developed from the traditional one in order to overcome its successive, time consuming, and dependent sequential phases, as well as reducing the contracting activities by the owner and gaining early involvement by the contractor [9], [18]. In the DB delivery method, figure 2, the owner contracts with one entity (Designer and Builder) which handles both designing and constructing the project, hence one contract is signed by the owner. Furthermore, the applicability and practicality of the final design is enhanced due to early engagement of the contractor [1], [21]. Usually, a lump sum (LS), guaranteed maximum price (GMP), or target price (TP) contract sum is committed with the DB.

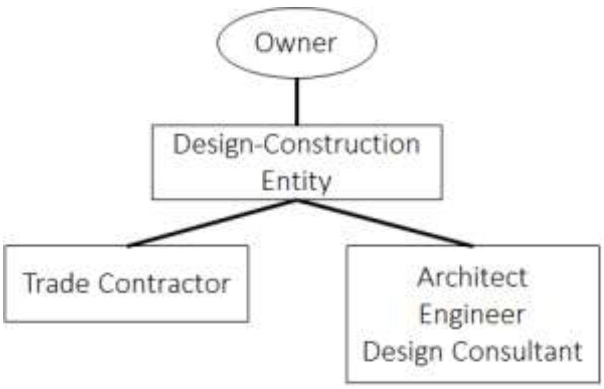

Fig. 2: Schematic representation of the Design-Build (DB).

Construction management at risk (CM@R)

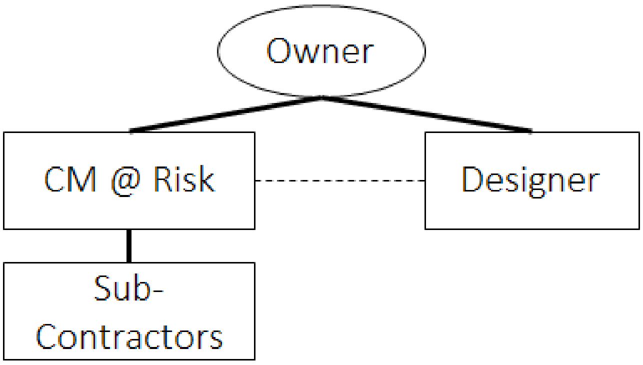

Fig. 3: Schematic representation of the Construction Management at Risk (CM@R).

A delivery method in which the risks and responsibilities are transferred to a construction management entity, which guarantees the design serviceability and construction performance to deliver the project within a defined time and cost as well as determined quality [9], [18], [21]. The CM@R entity, figure 3, provides the owner with design and construction consultancy services during design phase and becomes the general contractor during construction phase. Fixed lump sum (FLS) or guaranteed maximum price (GMP) usually committed in the (CM@R).

Integrated project delivery (IPD)

A project delivery method, figure 4, which integrates the services, efforts and expertise of the project stakeholders in order to develop an overall and common vision of the project as well as establishing plans and execution mechanism for joint and communal work strategy [28], [29]. The owner selects the project team which gathers and meets with the stakeholders of the project in order to clarify the vision as well as sharing and transferring expertise, suggestions and opinions for the beneficial advantages and unanimous decision making purposes. Early involvement and productive engagement of the stakeholders in the early stages of planning helps in anticipating and preventing future obstacles, such as creeping costs, and slipping schedules. However, due to its strategic and broad-spectrum nature, IPD can include different delivery methods with differing levels of involvement and varying timings of engagement [9], [18], [29]. Somehow new in the construction industry, the IPD has not been used extensively in the industry and its success depends on the level of experience and the ability of teamwork of the contracting parties [1], [28].

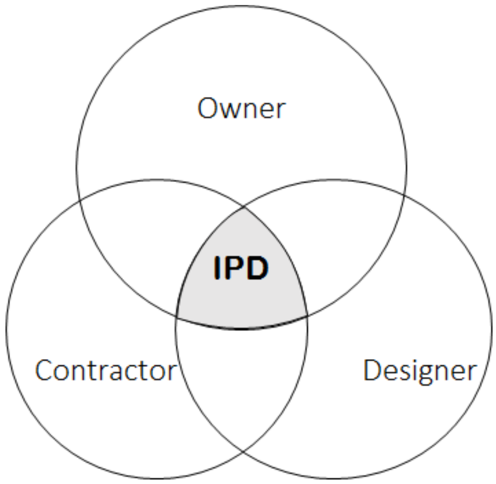

Fig. 4: Schematic representation of the Integrated Project Delivery (IPD).

B. Selection criteria

Numerous delivery methods have been used in the construction industry, which critically effect the success of the project [19]. Different projects and circumstances as well as different preferences lead to different delivery method and Not all selection factors are suitable for all projects [8], [9], [12]–[15], [18], [20], [25], [26]. The appropriateness of the delivery method depends on many factors, namely project nature, criticality and characteristics and owner experience, objectives and preferences as well as risk allocation and external factors [19], [26], [30]. However, the selection criteria vary considerably from project to project, furthermore, the significance of each criterion differs substantially from owner to owner [5], [7], [9], [12], [15], [20], [22].

Ali Hosseini et al. [15] identified 22 criteria of the most used in the construction industry for selecting the appropriate project delivery method. However, due to the uniqueness of each project as well as the different preferences of owners and internal and external circumstances, the list of the aforementioned 22 criteria would change and the significance of each criterion will depend on the specific project.

QingPing Zhong et al. [9] published a research provided detailed review and appraising the factors considered in the selection of PDM. The research presented a definitive and comprehensive list of factors impacting project delivery selection and provided general but critical factors to be considered when selecting the construction project delivery method, including: time, cost, quality, risk, project scope, owner’s issues, contractor’s issues, resource availability and quality, external market factors, and corresponding sub-criteria [2], [4], [5], [9].

For the main purpose of the current research and in order to develop a simple yet effective model, the following four factors have been used as selection criteria (see table 1):

Table 1: Selection criteria.

Criteria	**Definition**
Project Scope/Quality	Quality and scope of the project as stated in the contract. The contracting parties; depending on the project delivery method and legal obligations, have to agree and guarantee the successful completion of the project with the

	stated scope and quality.
Project Time	The contracted completion date of the project. The contracting parties; depending on the project delivery method and legal obligations, have to agree and guarantee the successful completion of the project on a specific date.
Project Cost	The contracted cost of the project. The contracting parties; depending on the project delivery method and legal obligations, have to agree and guarantee the successful completion of the project within predetermined budget.
Project Risks	Risks associated with the implementation and management of the project and the uncertainties of the project internal and external environment, as well as risks caused by the discreteness of work and organizational arrangement from the delivery method.

C. Decision-making method (DMM)

In recent times, the use of multi-criteria decision making (MCDM) in the construction industry, namely alternative selection, has become the main focus of research and a sought-after topic due to the importance of criteria and the criticality of decisions [3], [6], [23]. One of the most common and widely used method is the Analytic Hierarchy Process (AHP), see figure 5. AHP is a multi-criteria quantitative evaluation method which based on a pairwise comparison for determining the weights of criteria and the scores of alternatives [6], [31], [31]–[35]. The analytic Hierarchy Process (AHP) is considered one of the decision making methods that proof to be both comprehensive and simple, a well-known and universally used method which based on multi-criteria decision making theory [6]. Due to its widespread use and diverse applicability, the AHP is used in this research as a comparison tool for determining weights of criteria and a decision technique for selecting the optimal alternative.
Broadly speaking, the AHP is a theoretical and methodological approach for relative measurement [32], [33], [35]–[37], in which, we are not interested in the exact measurement of some quantity, but rather on the proportions between them. Hence, the AHP provides a pairwise comparison between two items, furthermore, a multiple comparison can be derived by hierarchically crossing alternatives with each criterion, figure 5. The Analytic Hierarchy Process (AHP) (Saaty, 1977 and 1980) is a multiple criterion evaluation methodology that is both descriptive and prescriptive, it can be used as a method of measurement with ratio scales [31], [37]. The Analytic Hierarchy AHP does not prescribe that judgments be perfectly consistent, nor does it prescribe when or when not to allow for rank reversals. AHP allows the decision makers to decide how much inconsistency is reasonable, if any, and whether nor not rank reversal (a reflection of relative rather absolute worth) should be permitted [31], [32], [37].
In this decision-making approach, figure 5, a decision tree is developed by, firstly, defining selection criteria and decision preferences, and the pairwise comparison is used to determine the relative weights of each pair of decision criteria, secondly, determining available alternatives, and the pairwise comparison is used in determining the comparative proportions of alternatives regarding each criterion [31]–[33], [35]–[37].

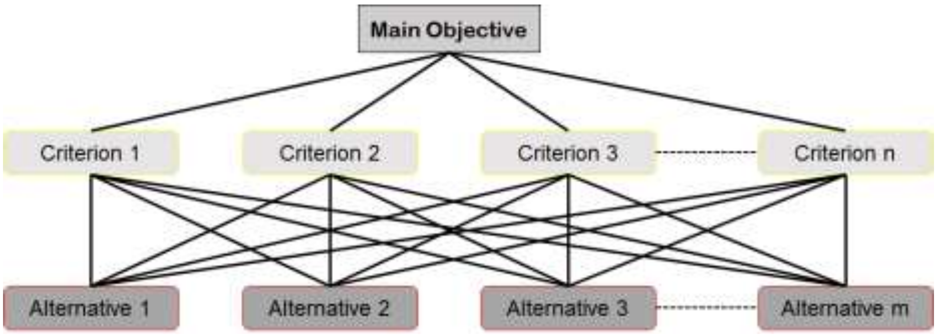

Fig. 5: Schematic representation of the Analytic Hierarchy Process Tree (AHP).

The final score of each alternative is calculated by multiplying comparative proportions of each alternative by the relative weight of the corresponding criterion. Ranking of the alternatives by the final scores indicates the selection order of alternatives regardless of the exact values of the scores, and an optimal alternative is selected depending on the criteria preferences. Decomposing the problem into two basic and simplified hierarchical levels, namely; criteria and alternatives, separately analysed and then combined by multiplication arrangement in a one measurement for each alternative. Whether extracted from exact measurements or rough estimations, the hierarchy will produce the optimal alternative for the final decision as long as the judgements about relative importance of criteria and alternatives are consistent and rational.

Regardless of the correctness of the outcomes, the AHP delivers an acceptable ranking of the alternatives which helps the decision makers in finding the best solution for the specific problem, a comprehensive and rational framework for decision making. Qualitative evaluation is possible by quantitatively relating criteria and alternatives with judgemental measurements.

III. DEVELOPMENT OF THE MATHEMATICAL MODEL

The mathematical model developed herein utilizes The analytic hierarchy process (AHP) as a decision tool. For a well-known and recognized technique, it is needless to explain the steps entailed by the AHP. The following are the steps required to develop the mathematical model using the analytic hierarchy process:

A. Professional expertise and academic knowledge On PDMs regarding criteria

The expertise of the five professionals and the knowledge of three academics in the field of construction industry regarding construction project delivery methods have been collected and tabulated. The collected data had had been investigated and screened in order to explore the orientation and guarantee the affinity of judgements. In order to obtain unanimous judgements, the individual judgments that are incompatible with and opposing the orientation of the majority are neglected, moreover, outliers have been eliminated. The orientations of the majority, $\geq 60\%$ of data, of judgements were considered the mainstream and conventional opinions and their averages were calculated and generalized as unanimous judgements.

The steps involved in collecting, screening and normalizing the data are as follows, check the accompanying illustration which used in the questionnaire as a selection table:

Criterion 1	9	7	5	3	1	3	5	7	9	Criterion 2

a) Collecting data by means of especially designed questionnaire. Explaining the coding system and number significance, as well as the pairwise relationship between two criteria to the experts.
b) In order to normalize and standardise the orientation of judgements; favouring the right or left side of the number one, the researcher assumed a majority of 60% of judgements, e.g., if at least five out of the eight experts gave judgements left to one (3,5,7,9), then the orientation would be favouring criterion 1 over criterion 2, otherwise, the orientation would be favouring criterion 2 over criterion 1. And the average of that orientation would be calculated, rounded to the closest integer and generalized.
c) If each four favoured different sides; i.e. four left and four right, the averages of each group would be calculated, rounded to the closest integer and the orientation of the higher average would be adopted along with the higher average. (the researcher did not encounter such case)

However, luckily, the judgements of the professionals and academics were similarly and commonly oriented. The generalized unanimous judgements were returned and presented to the experts and academics who in turn accepted the modified versions of judgements and considered them as a more rational and coherent evaluations of judgements.
The following tables, tables 2 to 5, are the final judgements of pairwise comparisons between the alternatives regarding each criterion.

Table 2: Pairwise comparisons between the alternatives regarding scope/quality.

Scope/Quality	DBB	DB	CM@R	IPD
DBB	1	1/3	1/7	1/5
DB	3	1	1/5	1
CM@R	7	5	1	5
IPD	5	1	1/5	1

Table 3: Pairwise comparisons between the alternatives regarding duration.

Duration	DBB	DB	CM@R	IPD
DBB	1	1/7	1/7	1/5
DB	7	1	3	5
CM@R	7	1/3	1	3
IPD	5	1/5	1/3	1

Table 4: Pairwise comparisons between the alternatives regarding cost.

Cost	DBB	DB	CM@R	IPD
DBB	1	5	7	3
DB	1/5	1	5	1/3
CM@R	1/7	1/5	1	1/5
IPD	1/3	3	5	1

Table 5: Pairwise comparisons between the alternatives regarding risks.

Risks	DBB	DB	CM@R	IPD
DBB	1	3	1/7	1/5
DB	1/3	1	1/9	1/7
CM@R	7	9	1	3
IPD	5	7	1/3	1

B. Calculating the final scores of each PDM regarding each criterion

Utilizing pairwise comparison and applying the AHP calculations, the final scores of each PDM regarding each criterion are as follows (table 6):

Table 6: Scores of each PDM regarding each criterion.

Criteria	DBB	DB	CM@R	IPD
Scope/Quality	0.05	0.15	0.62	0.17
Duration	0.04	0.55	0.28	0.13
Cost	0.56	0.13	0.05	0.26
Risks	0.08	0.04	0.58	0.29

According to Saaty [32], in practice, matrices with values $CR \leq 0.1$ are accepted. Fortunately, the consistency ratios of the four matrices were $0.06, 0.09, 0.09, and\ 0.06$, respectively. Hence, the four matrices are consistent and the final scores in table 6 are acceptable and considered satisfactory for making decisions.

C. The mathematical model

The generalized mathematical model is a summation of the multiplications of the final score of each PDM regarding each criterion and the preference weight of that criterion. Four equations will be generalized, one for each PDM, as follows:

$$\mathrm{DBB} = 0.05\mathrm{SQ} + 0.04\mathrm{D} + 0.56\mathrm{C} + 0.08\mathrm{R} \quad (1)$$
$$\mathrm{DB} = 0.15\mathrm{SQ} + 0.55\mathrm{D} + 0.13\mathrm{C} + 0.04\mathrm{R} \quad (2)$$
$$\mathrm{CM@R} = 0.62\mathrm{SQ} + 0.28\mathrm{D} + 0.05\mathrm{C} + 0.58\mathrm{R} \quad (3)$$
$$\mathrm{IPD} = 0.17\mathrm{SQ} + 0.13\mathrm{D} + 0.26\mathrm{C} + 0.29\mathrm{R} \quad (4)$$

Where
SQ: Scope and quality preference
D: Duration preference
C: Cost preference
R: Risk preference
Note that, $\mathrm{SQ} + \mathrm{D} + \mathrm{C} + \mathrm{R} = 1$.

IV. CASE STUDY

A project of constructing an intercity highway in Libya was used as a case study for selecting the project delivery method that best serves the preferences of the owner regarding the established four criteria. The project is a proposed highway connecting the two cities

Misrata and Tripoli by a coastal road of about 250 kilometres. For the present time; the year 2023, the project is in the conceptual stage, and yet to be planned and designed, which make the selection of the delivery method a high necessity and presently convenient to get full advantages of the selected method and to achieve full involvement by the project parties. A professional with over twenty years of experience in public construction sector in Libya was considered for representing the owner, Libyan government, as a representative, the selection criteria were offered to the professional in order to obtain preference weights of the objective criteria.
At the stage of selecting the delivery method, the owner is yet to contract with a designer nor contractor, and due to this early stage, the owner has minimum idea about the project. However, the general preferences regarding scope and quality, cost, duration and risks can be roughly estimated for the selection of the delivery method.

A. Preferences matrix

Presenting the four criteria, scope/quality, duration, cost and risks, and describing the meaning and significance of each, the owner's representative can prioritize his preferences arbitrarily by assigning weights to each criterion, with a total of 100%. However, pairwise comparison was conducted and criteria weights were calculated accordingly. Table 7 gives the pairwise comparisons between the criteria regarding main objective.

Table 7: Pairwise comparisons between the criteria regarding main objective.

Criteria	Scope/Quality	Duration	Cost	Risks
Scope/Quality	1	5	7	1
Duration	1/5	1	1	1/7
Cost	1/7	1	1	1/3
Risks	1	7	3	1

B. Preference vector

Utilizing pairwise comparison and applying the AHP calculations, the criteria preferences, represented as a preference vector in table 8, are as follows:

Table 8: Preferences vector.

Criteria	Preference
Scope/Quality	0.45
Duration	0.08
Cost	0.09
Risks	0.39

Again, according to Saaty [32], in practice, matrices with values $CR \leq 0.1$ are accepted. Fortunately, the consistency ratio of the pairwise comparisons matrix between the criteria regarding main objective was 0.04. Hence, the matrix is consistent and the final weights in table 8 are acceptable and considered satisfactory for making decisions.

C. Selecting the appropriate project delivery method

The selection calculations can be summarized in the following table:

Table 9: Selection table.

Criteria	Preference	DBB	DB	CM@R	IPD
Scope/Quality	0.45	0.05	0.15	0.62	0.17
Duration	0.08	0.04	0.55	0.28	0.13
Cost	0.09	0.56	0.13	0.05	0.26
Risks	0.39	0.08	0.04	0.58	0.29

Multiplication of score value of each PDM in relevant criteria by the weight of the corresponding criteria, the total score of each project delivery method is obtained and consequently, ordering of the project delivery methods by final scores and selecting the one with the highest score.

D. Final scores

The total score of each delivery method is calculated and the final ranking is presented in the following table, see table 10.

Table 10: Final scores of the project delivery methods.

PDM	Score
CM@R	0.53
IPD	0.22
DB	0.14
DBB	0.11

From table 10, construction management at risk (CM@R) is the appropriate and optimal method for delivering the case study project under the preferences presented in table 8. CM@R scored a high total of 0.53 which makes it very preferable and likely to satisfy the owner, hence the success of the project. Furthermore, integrated project delivery (IPD) scored second with low total of 0.22 which is lower than half the score of CM@R making the IPD inappropriate for application depending on the case study requirements. Lastly, both design-build (DB) and design-bid-build (DBB) scored very low with 0.14 and 0.11, respectively, hence both are not suitable for delivering the project under the current preferences.

V. RESULTS AND DISCUSSION

The current research proposed a mathematical model for selecting the method to be applied for delivering construction projects. Regardless of the exact values, the final scores reflect the order of the four delivery methods depending on an individual criterion as well as combinations of the four criteria.

A. Order of PDMs depending on an individual criterion

From table 6, the order of the four PDM can be visualized for each criterion. A selection can be made visually when considering individual criteria; when prioritizing the four alternatives depending on just one criterion, the alternative with the highest score related to that criterion is selected, followed in an orderly fashion the alternative with the next highest score, and so on.

Depending on the scope/quality of the project, the CM@R ranked first as the best alternative for delivering a project with the stated scope and quality, followed by IPD in the second rank and DB and DBB in the third and fourth ranks, respectively, which is the expected order as research proved and experience showed. Depending on the duration of the project, DB ranked first as the best alternative for delivering the project on a specific date, followed by CM@R in the second rank, and IPD and DBB ranked third and fourth, respectively, again, this order is expected academically and practically. Depending on the cost of the project, DBB ranked first as the best alternative for delivering the project within predetermined budget, followed by IPD in the second rank and DB and CM@R in the third and fourth ranks, respectively, which is academically and practically the expected order. Depending on the risks surrounding the project, CM@R is the best alternative for delivering the project with minimum

exposure to risks, followed by IPD in the second rank and DBB in the third rank, with DB in the fourth rank.

The final decision for selecting the PDM, when considering one criterion, can be made using table 11. For example, when considering scope/quality of the project as a top priority, the CM@R is the best choice, however, if the CM@R was not an option, the IPD is the one to be selected, and so on for the same criterion as well as the other criteria.

The following table summarizes the best choice of PDMs for application regarding each criterion.

Table 11: Choice order of PDMs depending each criterion.

Criterion	First choice	Second choice	Third choice	Fourth choice
Scope/Quality	CM@R	IPD	DB	DBB
Duration	DB	CM@R	IPD	DBB
Cost	DBB	IPD	DB	CM@R
Risks	CM@R	IPD	DBB	DB

Figure 6 shows the ranking of the PDMs depending on each criterion.

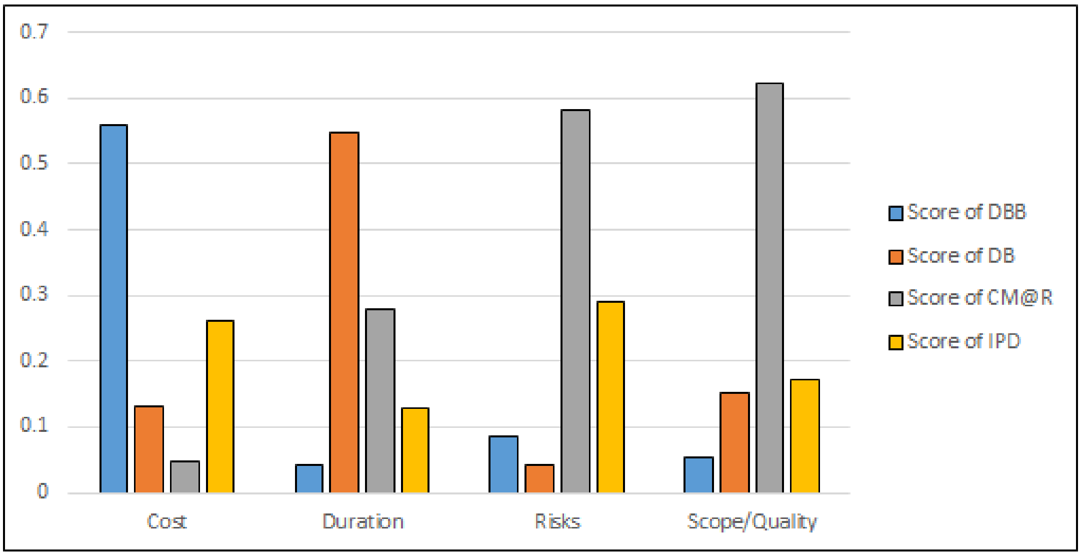

Fig. 6: Bar chart visualization of the ranking of the PDMs depending on each criterion.

The following table summarizes and categorizes the frequency of order in each rank for each PDM, see table 12.

Table 12: Summary of the frequency of order in each rank for each PDM with the related categories.

	First Rank		Second Rank		Third Rank		Fourth Rank	
PDM	Frequency	Category	Frequency	Category	Frequency	Category	Frequency	Category
CM@R	2	Scope/Quality Risks	1	Duration	-	-	1	Cost
DB	1	Duration	-	-	2	Scope/Quality Cost	1	Risks
IPD	-	-	3	Scope/Quality Cost Risks	1	Duration	-	-
DBB	1	Cost	-	-	1	Risks	2	Scope/Quality Duration

B. Order of PDMs depending on the four criteria

The following table shows the overall score of each PDM as a summation of the scores regarding each criterion:

Table 13: Total scores of the project delivery methods.

Criteria	DBB	DB	CM@R	IPD
Scope/Quality	0.05	0.15	0.62	0.17
Duration	0.04	0.55	0.28	0.13
Cost	0.56	0.13	0.05	0.26
Risks	0.08	0.04	0.58	0.29
Total	0.74	0.87	1.53	0.85

As an overall ranking of the PDMs with respect to the all four criteria, presuming equal preferences of the four criteria, CM@R ranked the first with total of 1.53, followed by DB with total score of 0.87. IPD was the third with total score of 0.85, and lastly DBB with 0.74 in total. As indicators, the numbers 0.74, 0.87, 1.53 and 0.85 show the order of the four PDMs in the totality of criteria as equal importance. Figure 7 is a bar chart representation of the PDMs in the totality of criteria as equal importance.

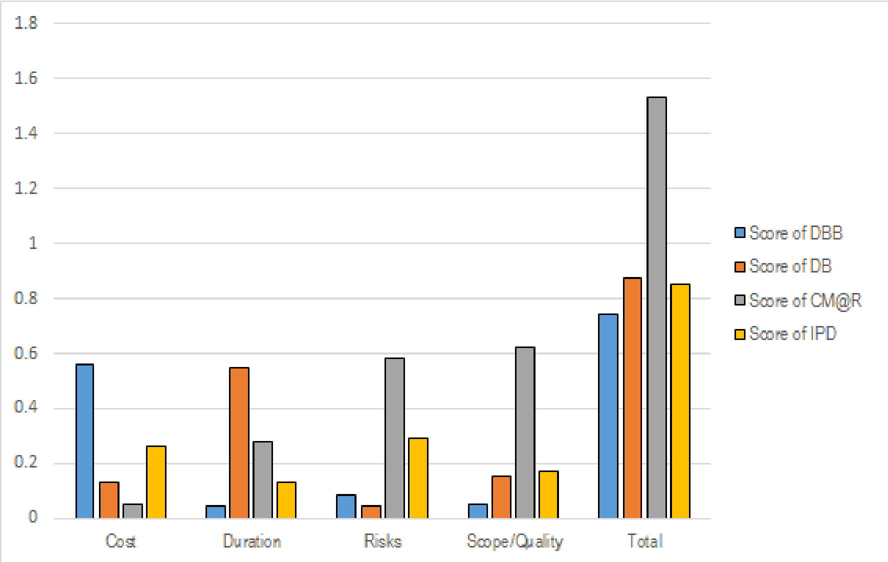

Fig. 7: Bar chart visualization of the ranking of the PDMs depending on each criterion and the totality of criteria as equal importance.

C. Case study results

It is widely recognised and internationally known that there is no “best” delivery method for a construction project [5], [8], [20], [26]. Moreover, it is evident that the appropriate method depends noticeably on the preferences of the owner [8], and inevitably, changes from project to project [9], [22], [30].

Interpreting and understanding the preferences of the owner of the case study project, referring to table 10, the order of preference of the four methods for delivering the case study project is evident.

Obviously that CM@R is the most preferred method, the CM@R scored 0.62 and 0.58 in the criteria scope/quality and risks, respectively, which have the highest weights of preferences by the owner, 0.45 and 0.39, respectively, hence by multiplying the scores by the related weights, the overall score is the highest for CM@R. Such results reflect previous findings regarding this method in terms of project scope, quality and risks [9], [21]

However, the IPD was the second preferred method to be implemented in the case study, aligning research findings with the literature [9], [29]. However, due to the huge difference in the scores between CM@R and IPD in the criteria scope/quality and risks, the total score of IPD was lower than the half of CM@R, rendering the IPD inappropriate for delivering the case study project.

Regarding the DB and DBB, both scored the highest in duration and cost, 0.55 and 0.56, respectively, which aligns with previous research findings [1], [9], [21] however, due to the disinterest of the owner in both criteria, preferred 0.08 and 0.09, respectively, their final scores were as low as 0.14 and 0.11, respectively, rendering them unacceptable for the case study.

VI. CONCLUSIONS

The current research presents a simple method for selecting the project delivery method based on the four criteria; scope/quality, duration, cost and risks. The researcher considered and investigated four of the common project delivery methods, as well as proposing four selection criteria based on the previous studies and experience. A prioritization technique was developed by using the analytic hierarchy process, and four equations were established for calculating the final score of each PDM depending on the four proposed criteria, the four equations (1, 2, 3, 4) are:

$DBB = 0.05SQ + 0.04D + 0.56C + 0.08R$

$DB = 0.15SQ + 0.55D + 0.13C + 0.04R$

$CM@R = 0.62SQ + 0.28D + 0.05C + 0.58R$

$IPD = 0.17SQ + 0.13D + 0.26C + 0.29R$

Which can be used simply by substituting the four variables, SQ, D, C and R, either arbitrarily or by applying any desired technique. The current research relied on the analytic hierarchy process for both calculating the scores of alternatives regarding each criterion and the weight of each criterion.

Table 14: The generalized matrix of the proposed mathematical model.

Criteria	Preference	DBB	DB	CM@R	IPD
Scope/Quality	SQ	0.05	0.15	0.62	0.17
Duration	D	0.04	0.55	0.28	0.13
Cost	C	0.56	0.13	0.05	0.26
Risks	R	0.08	0.04	0.58	0.29

A logical and rational decisions can be made using the developed model which can be used as a support tool in decision making. Regardless of the correctness of the final scores, the model provides a good indication about the behaviour of the PDMs when considering the four criteria, as well as a balanced scale when comparing the alternatives on the basis of pairwise comparisons. The four equations (1, 2, 3, 4) can be provided in a matrix form with preferences as variables, as shown in table 14. By which, selecting delivery method, after providing the four criteria preferences, can be done easily and simply by vector multiplications; preferences vector by PDM vector to obtain the overall score of the PDM. Finally, the process of developing the model revealed a ranking order of the four delivery methods when considering individual criteria of the considered four criteria, when considering one criterion, the project delivery methods can be ranked categorically, which, in turn, can help decision makers in when selecting the PDM. The categorical order of the project delivery methods is presented in the following table.

Table 15: The proposed categorical order matrix.

	Ranking			
Category	First Rank	Second Rank	Third Rank	Fourth Rank
Scope/Quality	CM@R	IPD	DB	DBB
Duration	DB	CM@R	IPD	DBB
Cost	DBB	IPD	DB	CM@R
Risks	CM@R	IPD	DBB	DB

I. RECOMMENDATIONS AND FUTURE STUDIES

The current research utilized the analytic hierarchy process as a decision tool, which considered a sufficient technique for the purposes of the current work, as well as nominating four project delivery methods and four selection criteria, however, future research should be done utilizing more sophisticated techniques such as genetic algorithms and neural networks, as well as investigating and considering more delivery methods and additional selection criteria and sub-criteria.

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Development of a mathematical model for the selection of the Project Delivery Method for the Libyan Public Construction Projects

تطويرنموذج رياضي لإختيار طريقة تسليم مشاريع الإنشاء في القطاع العام الليبي

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ملخص البحث:
إختيار طريقة تسليم المشروع (PDM) لتنفيذ أي مشروع إنشاء يعتبر أحد العوامل المهمة والحرجة التي يعتمد عليها نجاح تنفيذ المشروع. بالإضافة إلى ذلك، العملية التي سيتم بها إختيار طريقة تسليم المشروع يمكن أن تتنوع من قرارات بسيطة إلى عمليات معقدة ومتداخلة. يركز هذا البحث على تطوير نموذج رياضي بسيط لعملية الإختيار يستند على عملية التحليل الهرمي (AHP) كأداة قرار.
الأهذاف الأساسية من هذا البحث هي: تعريف صناعة الإنشاء المحلية بمفهوم تسليم المشروع؛ عرض الطرق الأكثر شيوعاً لتسليم المشروع؛ وتطوير نموذج بسيط وعملي ومفيد لإختيار الطريقة المثلي لتسليم مشروع معين.
توصل البحث إلى أن الطريقة المناسبة للإستخدام، بالإعتماد على تفضيلات المالك، هي إدارة الإنشاء في خطر (CM@R)، والتي حققت أعلى ترتيب بمجموع 0.53، تليها طريقة التسليم المتكامل (IPD) بمجموع 0.22. في حين حققت طريقتي تصميم-تنفيذ (DB) وتصميم-عطاء-تنفيذ (DBB) التراتيب الأخيرة بمجموع 0.14 و 0.11 على التوالي.

الكلمات المفتاحية: إدارة الإنشاء، مشاريع إنشاء الطرق، طريقة تسليم المشروع، تفضيلات القرار، عملية التحليل الهرمي (AHP).